Combinatoria y Optimización: Aplicación de Modelos Matemáticos Discretos

FQM371

University of Cádiz, Algeciras Campus, Spain

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INTERNATIONAL STAFF WEEK

Research and Technology Transfer at ASET

Our group - FQM371

Cádiz - 2023

Outline

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- AEI-DFG Directions on Distances in Graphs - bilateral project with Germany (in evaluation) - IGY and DK

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- Collaborations with many other universities. Spanish ones: Seville, URV (Tarragona), UPC. International: Many of them

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- Optimal design of discrete mathematical models and their applications.
- Applications of graph theory in computer science, social network privacy, and anything that would be possible.

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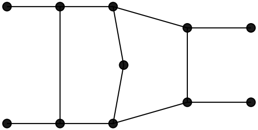

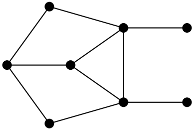

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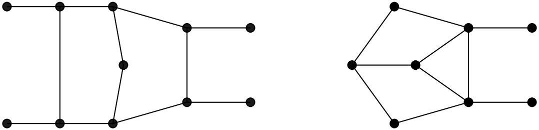

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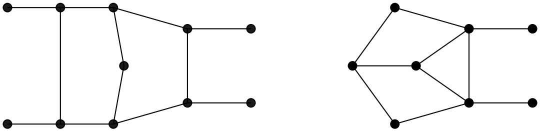

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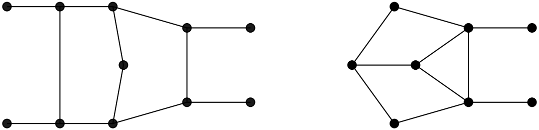

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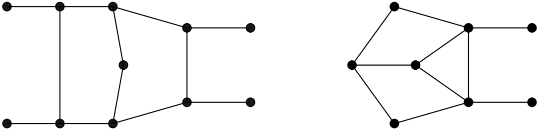

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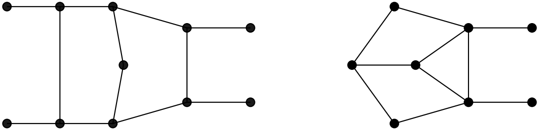

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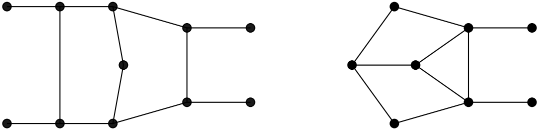

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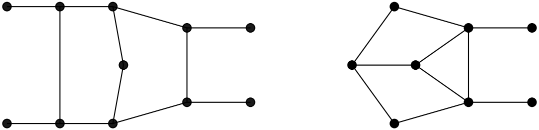

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- Computational complexity of such studies.

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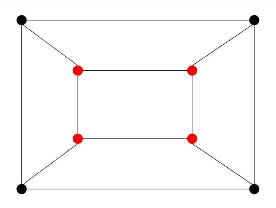
A dominating set in red



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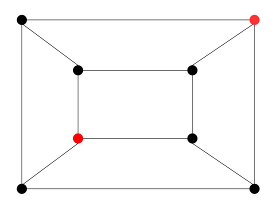

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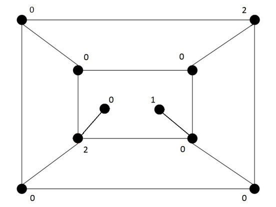
A Roman dominating function of minimum weight



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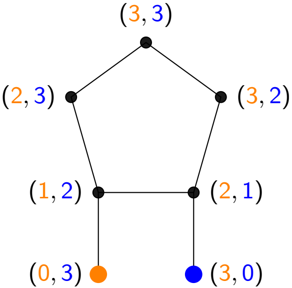

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- We have centered our attention in those attacks called "active attacks".

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- Establish links with some other nodes in the network (or even links between other nodes) in order to create some sort of "fingerprints" in the network.
- Once released the network, the adversary could retrieve the fingerprints already introduced, and use them to re-identify other nodes in the network.

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$adim_k(G)$: *k-metric antidimension* (minimum cardinality of a k-antiresolving set).

k-antiresolving sets ($k \geq 1$)

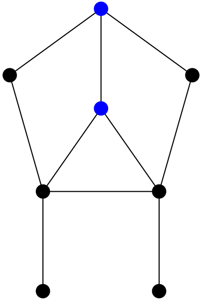
(Trujillo-Rasúa and IGYERO, 2016)

k-antiresolving set: set S s.t. for any vertex not in S there are at least $k-1$ other vertices not in S, not identified (through distances) by S **and** k **is the maximum possible**.

$adim_k(G)$: *k-metric antidimension* (minimum cardinality of a k-antiresolving set).

A 2-antiresolving set

(in blue)



k-antiresolving sets versus privacy

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- Adversary's (set S) background knowledge about a target (vertex v): metric representation of v with respect to S (ordered vector of distances)
- A set of attacker nodes S is a k-antiresolving set: the adversary cannot uniquely re-identify other nodes in the network with probability higher than $1/k$.

k-antiresolving sets versus privacy

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(k, ℓ)-anonymity

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G meets (k, ℓ)-*anonymity* with respect to active attacks, if k is the smallest positive integer such that the k-metric antidimension of G is lower than or equal to ℓ.

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- k is used as a privacy threshold, while ℓ is an upper bound on the expected number of attacker nodes in the network.
- ℓ can be estimated through statistical analysis (the number of attacker nodes is significantly lower than the total number of nodes in the network).

Outline

General position sets

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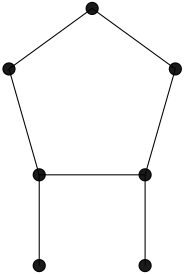

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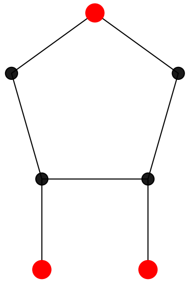

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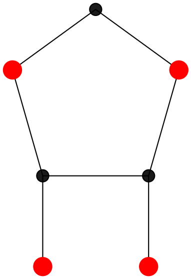

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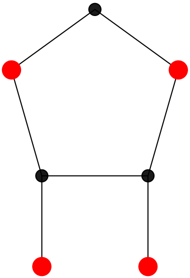

$\mathrm{gp}(G) = 4$

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- In the special case of hypercubes, the general position problem was studied back in 1995 by Korner related to some coding theory problem.
- J. Körner, On the extremal combinatorics of the Hamming space, J. Combin. Theory Ser. A 71 (1995) 112–126.

Outline

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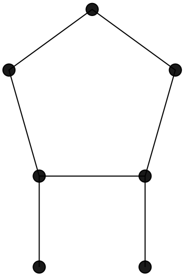

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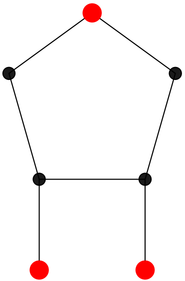

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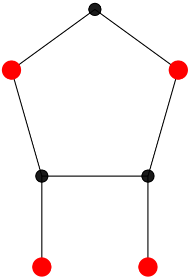

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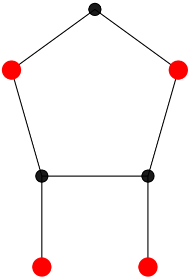

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- There is also $\mu_i(G)$ (independent version) and a few other ones.

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- Consider some possible variations of these parameters, which would lead to give more insight into other parameters of graphs.

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- There exists some possibilities for generating graphs with a known value for the metric dimension.

Thanks very much!!! We look forward any possible future collaboration.